



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

DIOPHANTINE ANALYSIS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

3. Proposed by O. S. KIBLER, Superintendent of Schools, West Middleburg, Logan County, Ohio.

It is required to find three whole numbers in an arithmetical progression, such that the sum of every two of them shall be a square.

- II. Solution by ARTEMAS MARTIN, A. M., Ph.D., LL.D., U. S. Coast and Geodetic Survey Office, Washington, D. C.

Let $x-y$, x and $x+y$ denote the required numbers in arithmetical progression. Then must

$$2x-y = \square = a^2 \dots (1), \quad 2x = \square = b^2 \dots (2), \quad 2x+y = \square = c^2 \dots (3).$$

Substituting $x = \frac{1}{2}b^2$, the value given by (2), in (1) and (3) we get
 $b^2 - y = \square = a^2 \dots (4)$, and $b^2 + y = \square = c^2 \dots (5)$.

From (4) and (5) we find

$$y = b^2 - a^2 = c^2 - b^2 \dots (6), \text{ therefore } 2b^2 = a^2 + c^2 \dots (7),$$

which is the only condition remaining to be satisfied.

Let $c = m + n$, and $a = m - n$, then (7) becomes

$$b^2 = m^2 + n^2 \dots (8),$$

which is satisfied by assuming $m = 2pq$, $n = p^2 - q^2$,

$$\text{Hence } x = \frac{1}{2}b^2 = \frac{1}{2}(m^2 + n^2) = \frac{1}{2}(p^2 + q^2)^2,$$

$$y = b^2 - a^2 = (m^2 + n^2) - (m - n)^2 = 2mn = 4pq(p^2 - q^2),$$

and the required numbers are

$$x - y = \frac{1}{2}(m^2 + n^2) - 2mn = \frac{1}{2}(p^2 + q^2)^2 - 4pq(p^2 - q^2),$$

$$x = \frac{1}{2}(m^2 + n^2) = \frac{1}{2}(p^2 + q^2)^2,$$

$$x + y = \frac{1}{2}(m^2 + n^2) + 2mn = \frac{1}{2}(p^2 + q^2)^2 + 4pq(p^2 - q^2).$$

Taking $p = 5$, $q = 4$, we get $x = 840\frac{1}{2}$, $y = 720$; hence $x - y = 120\frac{1}{2}$, $x + y = 1560\frac{1}{2}$, and multiplying by 4 for integers the required numbers are found to be 482, 3362 and 6242.

This set of numbers is the same as that found by different methods of solution in Maynard's edition of the key to Bonnycastle's Introduction to Algebra, published in London in 1835. See pp. 113-115.

An infinite number of other sets may be found.

4. Proposed by H. W. HOLYCROSS, Superintendent of Schools, Pottersburg, Union Co., Ohio.

What value of x will render $4x^4 + 12x^3 - 3x^2 - 2x + 1$ a square?

- II. Solution by ARTEMAS MARTIN, A. M., Ph.D., LL.D., U. S. Coast and Geodetic Survey Office, Washington, D. C.

$$\text{Put } 4x^4 + 12x^3 - 3x^2 - 2x + 1 = (2x^2 + 3x - 3)^2,$$

$$= 4x^4 + 12x^3 - 3x^2 - 18x + 9;$$

whence $x = \frac{1}{2}$. Other values may be found.

[Dr. Martin also sent excellent solutions to Nos. 1 and 2. R. H. Young, of West Sunbury, Pa., and Alvin E. Schmidt, Winesburg, Ohio, sent solutions to 1, 3

and 4. These solutions were not received in time to be acknowledged in March No.—ED.]

PROBLEMS.

9. Proposed by ISAAC L. BEVERAGE, Monterey, Virginia.

It is required to find three numbers the sum of whose 4th power is a square.

10. Proposed by L. B. HAYWARD, Bingham, Ohio.

Find two numbers such that each of them and also their sum and their difference when increased by unity shall all be square numbers.

AVERAGE AND PROBABILITY.

Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

2. Proposed by O. S. KIBLER, Superintendent of Schools, West Middleburg, Logan Co., Ohio.

Find the average area of a triangle formed by joining an angle of a square with any two points within the square.

Solution by Professor G. B. M. ZERR, Principal of High School, Staunton, Virginia.

Let $ABCD$ be the square side a , and U, V , the two random points.

Through V, U draw KM, NL , parallel to AD , KM meeting AU in E .

Let $AL = x$, $AK = w$, $LU = y$, $KV = z$, $KE = z'$.

Then $z' = \frac{wy}{x}$; also

area $AUV = \frac{1}{2}(wy - xz) = u$, when $z < z'$,

area $AUV = \frac{1}{2}(xz - wy) = u_1$, when $z > z'$.

The limits of x are 0 and a ; of w , 0 and x ; of y , 0 and a ; of z , 0 and z' , and z' and a .

Hence, the required average area is

$$\begin{aligned} \Delta &= \frac{\int_0^a \int_0^x \int_0^a \left\{ \int_0^{z'} u dz + \int_z^a u_1 dz \right\} dx dw dy}{\int_0^a \int_0^x \int_0^a dx dw dy dz} \\ &= \frac{2}{a^4} \int_0^a \int_0^x \int_0^a \left\{ \int_0^{z'} u dz + \int_z^a u_1 dz \right\} dx dw dy \\ &= \frac{1}{2a^4} \int_0^a \int_0^x \int_0^a \left(\frac{2w^2 y^2}{x} + a^2 x - 2aw y \right) dx dw dy, \end{aligned}$$

